Commutative algebra and algebraic geometry

Course for mathematicians and physicists
(MSc, Phd, advanced BSc level)

2024 autumn, BME

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Place and time: TBA, 2 x 2 hours/week

Description: Algebraic geometry originates in the study of solutions of systems of polynomial equations and their geometric properties. In the past few hundred years its task however became much more general. It has become a subject arching from number theory through complex manifolds to theoretical physics. Algebraic geometry, in a general sense, is one of the central areas of today's mathematics; the highest number of Fields medals were awarded for results related to algebraic geometry. In addition, since the second half of the twentieth century, algebraic geometry has important engineering and statistical applications.

The aim of this course is to cover the basics of algebraic geometry as well as the necessary commutative algebra required by it. The emphasis will be on the geometric view. We will get to know the basic properties of affine, projective, and abstract varieties and manifolds (e.g. dimensionality, irreducibility, Zariski topology, coordinate ring), and mappings between these. Important algebraic constructions will appear along the way, such as localization and categories.

Prerequisites: knowledge of basic abstract algebra (rings, fields, modules, homomorphisms, etc.). Knowledge of some differential geometry is beneficial.

Credit: 5

Grading: Homeworks (70%) + Two short midterms (30%)

Recommended textbooks:

- A. Gathmann: Algebraic geometry,
  https://www.mathematik.uni-kl.de/
  ~gathmann/class/alggeom-2021/alggeom-2021.pdf
- I. R. Shafarevich, Basic Algebraic Geometry I, Springer, 1994.
- M. Reid, Undergraduate Algebraic Geometry, Cambridge University Press, 1988.
- R. Hartshorne, Chapter 1 of Algebraic Geometry, Springer, 1977.
- M. Atiyah, I. Macdonald: Introduction to commutative algebra