Algebraic topology and homological algebra $_{ m Sheet 5}$ — Due 13/05

HW problems to hand in

1. Let R be a ring.

- (a) Show that the functor $-\otimes M$ commutes with (possibly infinite) direct sums.
- (b) Show that direct sums and direct summands of flat modules are flat
- (c) Show that the regular module R_R is flat.
- (d) Conclude that in R-Mod every projective module is flat.
- 2. For M an oriented closed connected n-manifold show that
 - (a) $H^n(M) = \mathbb{Z}$
 - (b) $H_{n-1}(M)$ has no torsion
 - (c) There exists a generator $\omega_M \in H^n(M)$ with $\omega_M([M]) = 1$

Hint: use the universal coefficient theorem and Poincaré duality