Topology Exercises 2

1 Knots

The *reflection* of a knot is the one we get when reflecting on a plane not intersecting the knot. If K is an oriented knot, the *reverse* of K is the same knot with the opposite orientation.

Show that the knot 4_1 (see Table 1) is equivalent to its reverse and reflection.

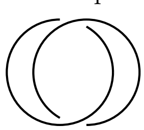
2 Writhe of a diagram

Compute the writhe of three diagrams appearing in the Table 1. (note that your answer may depend on the orientation you choose)

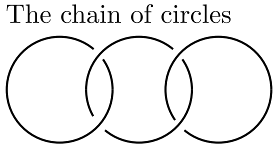
3 Jones polynomial

Compute the Jones polynomial of the following knots/links using the Kauffmann bracket identities.

- Trefoil
- The 4_1 knot (from Table 1)
- The Hopf link (1,1)



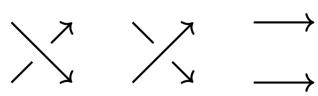
- The chain of circles



4 Skein identity

Compute the Jones polynomial via the Skein identity

$$tV_{L_+}(t) - t^{-1}V_{L_-}(t) = (t^{1/2} - t^{-1/2})V_{L_0}(t)$$



L_+ L_- L_0

for the following knots/links.

- Trefoil,
- The 4_1 knot (from Table 1)
- Hopf link (1,1)
- Chain of circles

5 Value of the Jones polynomial at $t = 1$

Prove that the value of the Jones polynomial of an oriented link $V_L(t)$ at $t = 1$ is $(-2)^{\#L-1}$ where $\#L$ denotes the number of connected components.

- Use the Skein relations to show that $V_L(1)$ does not change under crossing changes.
- Use the fact that you can get the trivial knot via crossing changes from any knot.
- Use the multiplicative property of the Kauffman bracket to show the statement.

6 Value of the Jones polynomial at $t = -1$

Prove that $V_{\bigcirc \cup L}(-1) = 0$ for any link L.

Figure 1: Table 1