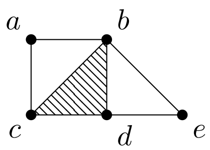
Computational topology
Sheet 10 — Homework

1. Let $K = \{v\}$ consist of a single vertex.
   (a) Compute the chain groups C_0 and C_1.
   (b) Compute the homology groups $H_0(K)$ and $H_1(K)$.
2. Let K consist of two vertices v_0, v_1 and one edge $[v_0v_1]$.
   (a) Compute the boundary map ∂_1.
   (b) Compute the homology groups $H_0(K)$ and $H_1(K)$.
3. Let K be the simplicial complex consisting of two disjoint edges.
   (a) Compute $H_0(K)$.
   (b) Interpret your answer in terms of connected components.
4. Let K be the boundary of a tetrahedron (all triangular faces included, but not the 3-simplex).
   (a) Describe the chain groups C_0, C_1, C_2.
   (b) Argue that $H_2(K) \cong \mathbb{Z}$.
   (c) Argue that $H_1(K) = 0$.
5. Let K contain a 2-simplex $[v_0v_1v_2]$.
   (a) Compute $\partial_2([v_0v_1v_2])$.
   (b) Explain how to recognize from a picture that a 1-cycle is a boundary.
6. Let K be any simplicial complex.
   (a) Show that if two vertices are connected by an edge, then they represent the same class in $H_0(K)$.
   (b) Conclude that $H_0(K)$ depends only on the connected components of K.
7. Let $c = \sum a_i[v_iv_{i+1}]$ be a 1-chain.
   (a) Write out the condition $\partial_1(c) = 0$.
   (b) Interpret this condition as a statement about coefficients at each vertex.
   (c) Explain why this corresponds to having "no endpoints."
8. Compute the simplicial homology of the following simplicial complex:



Budapest University of Technology and Economics, Hungary
Ádám Gyenge