Name:_____

Two-sided page. Each problem worths 10 points.

1. (a) Explain the difference between symbolic computation and numerical computation. Discuss one situation where symbolic computation gives structural insight that numerical computation cannot provide directly.
   (b) Explain the difference between exact rational arithmetic and floating-point arithmetic. Why can mathematically equivalent formulas produce different numerical results in floating-point computation?
   (c) In Python, every iterator is an iterable, but not every iterable is an iterator. Explain this statement and give one example involving generator expressions.

2. Consider the symbolic expression

$$f(x, y) = \frac{(x + y)^4 - (x - y)^4}{x}.$$

   Using SymPy, perform the following tasks:

   (a) Define the expression symbolically and simplify it.
   (b) Expand the numerator and determine whether the whole expression can be simplified to a polynomial.
   (c) Write a Python function that recursively traverses the expression tree and counts how many nodes of type `Pow`, `Add`, and `Mul` occur.
   (d) Determine the depth of the expression tree.
   (e) Substitute $y = x$ into the expression before simplification and explain why the order of substitution and simplification matters.

3. Solve the following tasks using SymPy.

   (a) Compute the first and second derivatives of

$$g(x) = x^3 \sin(x).$$

   (b) Compute the definite integral

$$\int_0^{\pi} x \sin(x)\, dx.$$

   (c) Compute the Taylor expansion of $\ln(1 + x)$ around $x = 0$ up to degree 6.
   (d) Solve the differential equation

$$y'' + 3y' - 4y = e^{2x}.$$

   (e) Verify symbolically that the obtained solution satisfies the equation.

4. Let

$$p(x) = x^5 - 2x^4 - x^3 + 2x^2, \qquad q(x) = x^4 - 1.$$

   Using SymPy, perform the following tasks:

(a) Factor both polynomials completely over the rational numbers.

(b) Determine whether $p(x)$ is divisible by $q(x)$. Compute quotient and remainder.

(c) Compute the greatest common divisor of $p(x)$ and $q(x)$.

(d) Compute all roots of $q(x)$, including complex roots.

(e) Rewrite $p(x)$ as a polynomial in powers of $(x - 1)$.

The End.